THE MATH TRANSLATOR

PRECALCULUS

CHAPTER 1

Practice Test Answer Key

1. $y = 2x + 8$

Yes, it is a function because it is a line and lines pass the Vertical Line Test.

2. $\{(2,1), (3,2), (-1,1), (0,-2)\}$

Yes, it is a function. Each input value leads to only one output value.

3. $f(x) = -3x^2 + 2x$

$$f(-2) = -3(-2)^2 + 2(-2)$$
$$= -3(4) - 4$$
$$= -12 - 4$$
$$= \boxed{-16}$$

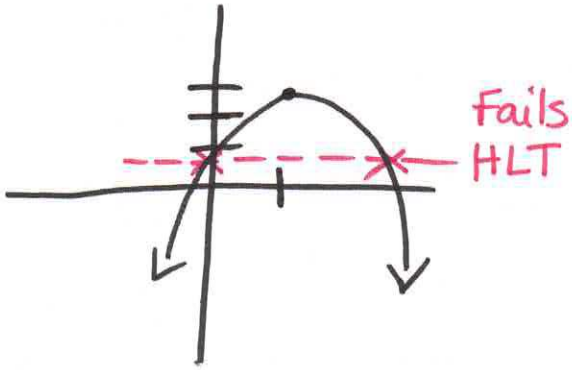
4. $f(x) = -3x^2 + 2x$

$$\boxed{f(a) = -3a^2 + 2a}$$

5. $f(x) = \underbrace{-2(x-1)^2+3}$

This is a parabola. Parabolas are not one-to-one because they fail the Horizontal Line Test.

Rough sketch of $f(x)$:



6. $f(x) = \sqrt{3-x}$

$$3 - \cancel{x} \geq 0$$
$$\underline{\cancel{+x}} \quad \underline{+x}$$

$$3 \geq x$$

or

$$x \leq 3$$

$$\boxed{\underline{\text{Domain}}: (-\infty, 3]}$$

7. $f(x) = 2x^2 - 5x$

$$f(a+1) = 2(a+1)^2 - 5(a+1)$$
$$= 2(a+1)(a+1) - 5a - 5$$
$$= 2(a^2 + 2a + 1) - 5a - 5$$
$$= 2a^2 + 4a + 2 - 5a - 5$$
$$= 2a^2 - a - 3$$

$$f(1) = 2(1)^2 - 5(1)$$
$$= 2(1) - 5$$
$$= 2 - 5$$
$$= -3$$

$$f(a+1) - f(1) = 2a^2 - a - 3 - (-3)$$
$$= 2a^2 - a - 3 + 3$$
$$= \boxed{2a^2 - a}$$

8. $f(x) = \begin{cases} x+1 & \text{if } -2 < x < 3 \\ -x & \text{if } x \geq 3 \end{cases}$

$f(x) = x+1$:

x	$f(x)$	
-2	-1	← open circle
0	1	
3	4	← open circle

$f(x) = -x$:

x	$f(x)$
3	-3
4	-4
5	-5

(work on next page)

$f(x) = x+1$:

$f(-2) = -2+1$
$= -1$

$f(0) = 0+1$
$= 1$

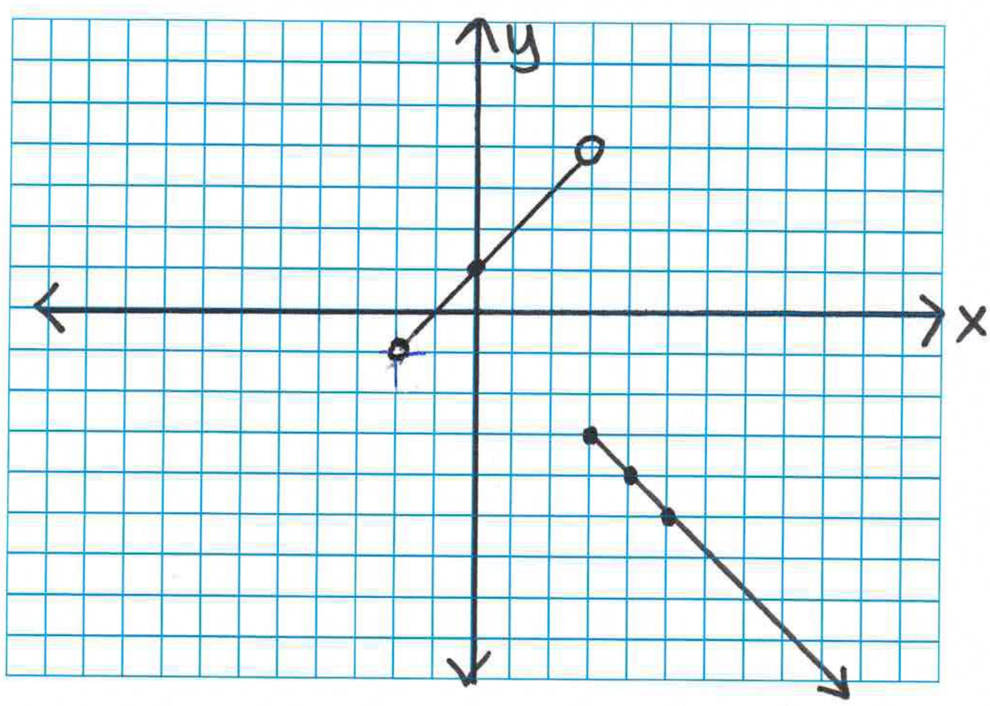
$f(3) = 3+1$
$= 4$

$f(x) = -x$:

$f(3) = -3$

$f(4) = -4$

$f(5) = -5$



9. $f(x) = 3 - 2x^2 + x$

$$\frac{f(b)-f(a)}{b-a} = \frac{3-2(b)^2+b-(3-2a^2+a)}{b-a}$$

$$= \frac{3-2b^2+b-3+2a^2-a}{b-a}$$

$$= \frac{-2b^2+b+2a^2-a}{b-a}$$

$$= \frac{-2b^2+2a^2+b-a}{b-a}$$

$$= \frac{-2(b^2-a^2)+b-a}{b-a}$$

$$= \frac{-2(b-a)(b+a)+(b-a)}{b-a}$$

$$= \frac{(b-a)(-2(b+a)+1)}{b-a}$$

$$= \frac{\cancel{(b-a)}(-2(b+a)+1)}{\cancel{b-a}}$$

$$= \boxed{-2(b+a)+1}$$

10. $f(x) = 3-2x^2+x$ and $g(x) = \sqrt{x}$

$(g \circ f)(x) = g(f(x)) = \boxed{\sqrt{3-2x^2+x}}$

11. $f(x) = 3-2x^2+x$ and $g(x) = \sqrt{x}$

$(g \circ f)(1) = g(f(1))$

$$f(1) = 3-2(1)^2+1$$
$$= 3-2+1$$
$$= 2$$

$$g(2) = \sqrt{2}$$

So, $\boxed{g(f(1)) = \sqrt{2}}$

12. $H(x) = \sqrt[3]{5x^2-3x}$

$(f \circ g)(x) = f(g(x)) = H(x)$

$$\boxed{f(x) = \sqrt[3]{x}\ ,\ g(x) = 5x^2-3x}$$

13. $f(x) = \sqrt{x+6}\ -1$

6 units left

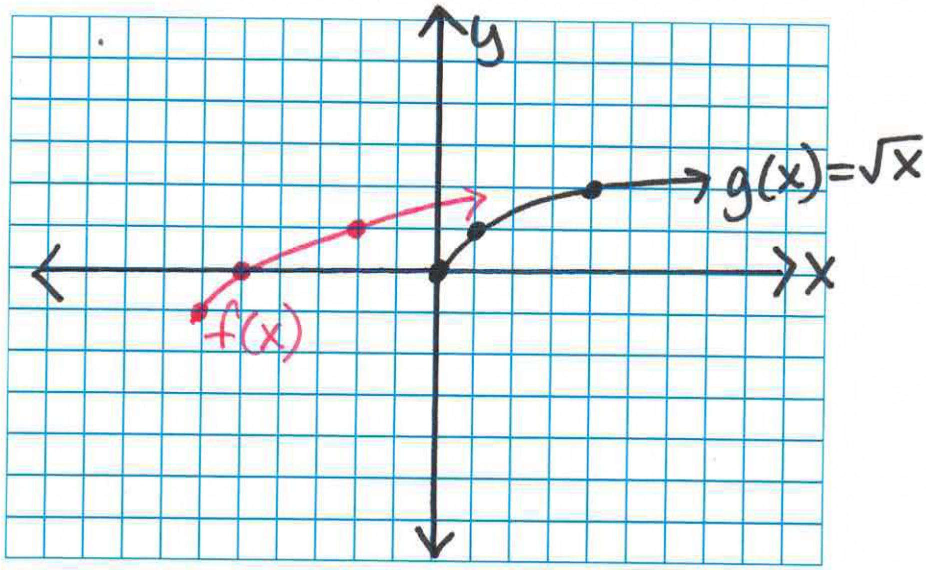
1 unit down

$g(x) = \sqrt{x}$ Key points: $\underbrace{(0,0)\ (1,1)\ (4,2)}$

Domain: $[0,\infty)$

$f(x) \rightarrow$ move 6 units left and 1 unit down

Graph on next page



(14.) $f(x) = \dfrac{1}{x+2} - 1$

1 unit down

2 units left

$g(x) = \dfrac{1}{x}$

Key points: $(-1,-1)$ $(1,1)$

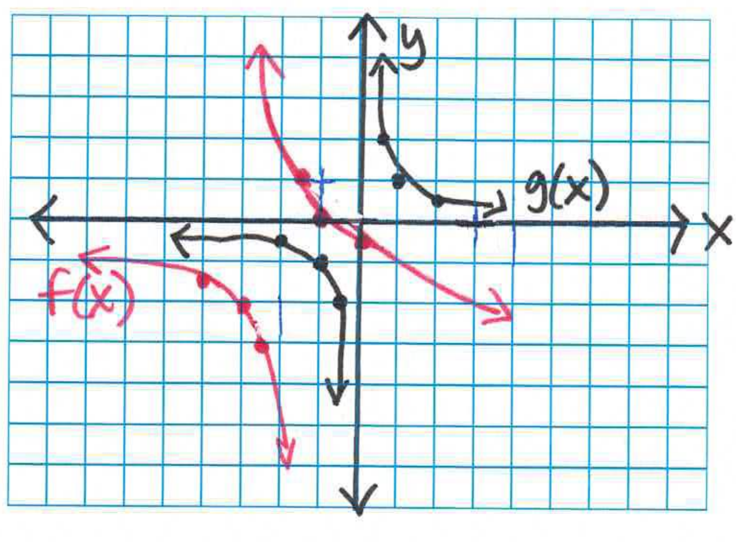
$(-2,-\frac{1}{2})$ $(2,\frac{1}{2})$

$(-\frac{1}{2},-2)$ $(\frac{1}{2},2)$

2 left, down 1

Domain:

$(-\infty,0)\cup(0,\infty)$



15. $f(x) = -\frac{5}{x^2} + 9x^6$

$$f(-x) = \frac{-5}{(-x)^2} + 9(-x)^6$$

$$= \frac{-5}{x^2} + 9x^6$$

Since $f(-x) = f(x)$, the function is $\boxed{\text{even}}$.

16. $f(x) = -\frac{5}{x^3} + 9x^5$

$$f(-x) = \frac{-5}{(-x)^3} + 9(-x)^5$$

$$= \frac{-5}{-x^3} - 9x^5$$

$$= \frac{5}{x^3} - 9x^5$$

$$= -f(x)$$

Since $f(-x) = -f(x)$, the function is $\boxed{\text{odd}}$.

(17.) $f(x) = \frac{1}{x}$

$f(x) = \frac{1}{-x}$

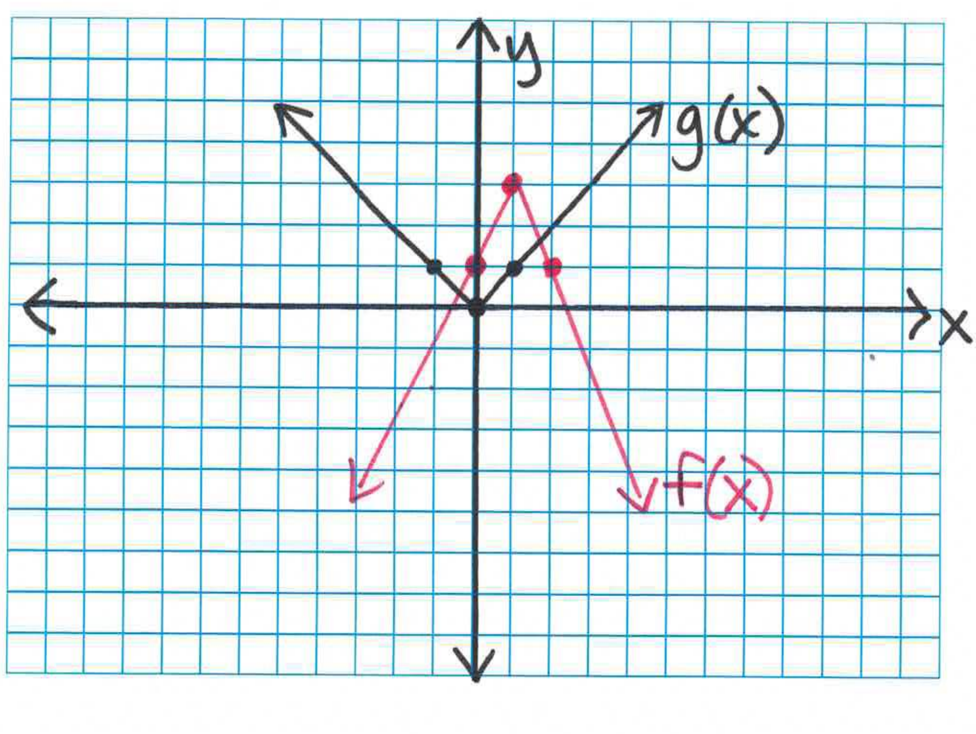
$f(x) = -f(x)$ which means $f(x)$ is $\boxed{\text{odd}}$.

(18.) $f(x) = -2|x-1|+3$

y by -2 right 1   up 3

$g(x) = |x|$ Key points: $(-1,1)$ $(0,0)$ $(1,1)$

y by -2: $\underbrace{(-1,-2)\ (0,0)\ (1,-2)}_{\text{right 1, up 3}}$



19. $|2x-3| = 17$

$$2x-\cancel{3}=17 \quad \text{or} \quad 2x-\cancel{3}=-17$$

$$\underline{\cancel{+3}} \quad \underline{+3} \qquad\qquad \underline{\cancel{+3}} \quad \underline{+3}$$

$$\frac{\cancel{2}x}{\cancel{2}}=\frac{20}{2} \qquad\qquad \frac{\cancel{2}x}{\cancel{2}}=\frac{-14}{2}$$

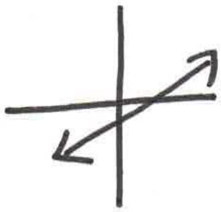
$$\boxed{x=10} \qquad\qquad \boxed{x=-7}$$

20. $$\frac{\cancel{-1}\left|\frac{1}{3}x-3\right|}{\cancel{-1}} \geq \frac{17}{-1}$$

$$\left|\frac{1}{3}x-3\right| \leq -17$$

Cannot apply the definition because the constant is negative. Logically, the absolute value of a number will never be less than a negative number. So there is $\boxed{\text{no solution}}$ here.

21. $f(x) = 3x - 5$



This is a line, so it is one-to-one

$$y = 3x - 5$$

$$x = 3y - \cancel{5}$$

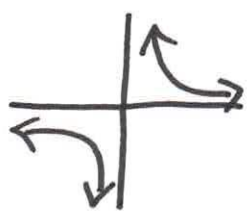
$$\underline{+5} \qquad \underline{+\cancel{5}}$$

$$\frac{x+5}{3} = \frac{\cancel{3}y}{\cancel{3}}$$

$$\frac{x+5}{3} = y$$

$$\boxed{f^{-1}(x) = \frac{x+5}{3}}$$

22. $f(x) = \dfrac{4}{x+7}$



This is a one-to-one function.

$$y = \frac{4}{x+7}$$

$$(y+7)(x) = \left(\frac{4}{\cancel{y+7}}\right)(\cancel{y+7})$$

$$x(y+7) = 4$$

$$\frac{\cancel{x}(y+7)}{\cancel{x}} = \frac{4}{x}$$

$$y + \cancel{7} = \frac{4}{x} - 7$$

$$\underline{-\cancel{7}}$$

$$y = \frac{4}{x} - 7$$

$$\boxed{f^{-1}(x) = \frac{4}{x} - 7}$$

23. Increasing on $(-\infty, -1.1)$ and $(1.1, \infty)$

24. Decreasing on $(-1.1, 1.1)$

25. Local minimum: $(1.1, -0.9)$

26. Local maximum: $(-1.1, 0.9)$

27. $f(2) = 2$

28. $f(-2) = 2$

29. $f(x) = \begin{cases} |x| & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$

30. $F(6) = 13$

31. $F(x) = 5$

$\uparrow y = 5$, so $\boxed{x = 2}$

32. The function is increasing on its domain (the y-values increase as the x-values increase).

33. Yes, each y-value corresponds to exactly one x-value.

34. $\underbrace{f^{-1}}(15)$

This is saying $y=15$, find the x that goes with it.

$$\boxed{f^{-1}(15)=7}$$

35. $f(x)=-2x+11$ This is a line so it is one-to-one.

$$y=-2x+11$$
$$x=-2y+\cancel{11}$$
$$\underline{-11} \quad \underline{\cancel{-11}}$$
$$\frac{x-11}{-2}=\frac{\cancel{-2}y}{\cancel{-2}}$$
$$\frac{x-11}{-2}=y$$

$$\boxed{f^{-1}(x)=\frac{x-11}{-2}}$$